EPL and capital-labor ratios

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May 6, 2013

Abstract

Employment protection (EPL) has a well known negative impact on labor flows as well as an ambiguous but often negative effect on employment. In contrast, its impact on capital accumulation and capital-labor ratio is less well understood. The available empirical evidence would suggest a non-monotonic relation between capital-labor ratios and EPL: positive at very low levels of EPL, and then negative.

We explore the theoretical effects of EPL on physical capital in a model of a firm facing labor frictions. Under standard assumptions, theory always implies a monotonic negative link between capital-labor ratios and EPL. For a positive link to arise, a very specific pattern of complementarity between capital and workers protected by EPL (senior workers, as opposed to unprotected new entrants, or junior workers) has to be assumed. Further, no standard production technology is able to reproduce the inverted U-shape pattern of the data.

An extension of the model with specific skills investment is instead able to reproduce the inverted U-shape pattern. EPL protects and therefore induces investments in specific skills. We calibrate the returns to seniority by using estimates from the empirical literature. Under complementarity between capital and specific human capital, physical capital and senior workers having accumulated specific human capital are de facto complement production factors and EPL may increase capital demand at the firm level. The paper concludes that labor market institutions sometimes have a positive role in a second-best environment.

\footnote{We are grateful to Jim Albrecht, Bruno Decreuse, Pieter Gautier and participants at the Workshop on Search and Matching in Chile. Alexandre Janiak acknowledges financial support by Fondecyt (project no 1120593).}
1 Introduction

An expanding literature investigates the role of employment regulations on labor market outcomes as well as other dimensions such as investment in physical capital, human capital, productivity, innovation and growth. Labor studies have shown that a particular component of these regulations, employment protection, has sizeable effects on unemployment rates, turnover, job flows and unemployment duration. Employment protection legislation (hereafter, EPL) has a well known negative impact on labor flows and an ambiguous but often negative effect on employment.1

The effect on investment and capital-labor ratio is less well understood and in the empirical literature, the effect of EPL is ambiguous. On the one hand, Autor et al. (2007) find that the effect may be positive in the US: the authors use the adoption of wrongful-discharge protections by U.S. state courts from the late 70s to the late 90s to evaluate the link between dismissal costs and other economic variables. With firm-level data, they find a positive effect of employment protection on capital-labor ratio. On the other hand, Cingano et al. (2010) find a negative effect on capital per worker in the case of European firms.2

In this paper we precisely analyse the effect of employment protection on capital accumulation and capital-labor ratio and attempt to reconcile those findings. The effect of EPL may be non-linear, and in particular, positive for investment at low values of EPL, and negative for investment at higher values, which would reconcile the various empirical findings of the papers cited in the previous paragraph. Without a claim on causality, Figure 1 at least suggests that an inverted U-shape pattern tends to emerge from the data: the x-axis is the standard OECD stringency index and the vertical axis is the capital-labor ratios. At low EPL level, the correlation is positive, and it becomes negative when the index becomes larger than 1.75. The regression analysis in Table 1 confirms this, but also the fragility of the correlation. The inclusion of a dummy variable for Anglo-Saxon countries leads to a negative and significant coefficient on EPL, while the inclusion of a dummy for “high EPL” countries produces a positive and non-significant correlation. Overall, the correlation coefficient is negative (-0.34) and in the regression, the linear effect is negative but not significant. The correlation coefficient is equal to 0.40 when Greece, Mexico, Portugal and Turkey are removed from the sample and takes the value -0.67 when Anglo-Saxon countries are not considered.

Our paper tries to make some theoretical sense about these results. We start by

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1 In influential papers have investigated the role of employment regulations on other dimensions such as productivity and growth: Hopenhayn and Rogerson (1993) and Bertola (1994) argue that productivity is lower because of a misallocation of employment to technologies, favoring less productive structures, leading to reduced incentives for capital accumulation. Bassanini et al. (2009) empirically document the link between employment protection and productivity growth and find that EPL reduces productivity growth in industries where EPL is more likely to be binding. There is also an emerging literature on the pattern of trade specialization: Saint-Paul 1997 and 2002 shows that countries with a rigid labor market will tend to produce relatively secure goods, at a late stage of their product life cycle and therefore innovate less, rather imitate. See also the more recent paper by Cuñat an Melitz (2010). There is a very active and broader literature extending models of trade to imperfect labor markets e.g., Costinot (2009).

2 Their methodology follows Rajan and Zingales (1998): it compares the impact on the demand for capital in sectors requiring large job reallocation with sectors where job reallocation is low.
investigating whether a theory of EPL and physical capital investment can generate these correlation patterns. Our results are that a standard model with capital and labor generally leads to a negative link between capital-labor ratios and EPL. It may generate a positive link, but this positive link is due to a specific pattern of complementarity between capital and workers protected by EPL (or senior workers, as opposed to unprotected new entrants, or junior workers): since EPL increases the share of senior workers in employment, high EPL leads to higher investment in physical capital given this complementarity. However, the positive link is monotonic over the whole range of EPL values: no standard production technology is able to reproduce the inverted U-shape pattern.\(^3\)

However, an extension of the model where workers invest in specific skills, is precisely able to reproduce this inverted U-shape pattern. The reason is that EPL protects and therefore induces investments in specific skills. Assuming complementarity between capital and specific human capital is natural and therefore, the complementarity between capital and senior workers becomes a natural outcome of the model:

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\(^3\)It is always possible to find a more complex production function leading to an inverted U-shape pattern but this would arguably be an artificial way of reproducing the empirical results.
Table 1: Regressions of capital-labor ratio on EPL stringency

<table>
<thead>
<tr>
<th>Specifications:</th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPL index</td>
<td>-0.088</td>
<td>-0.253***</td>
<td>0.079</td>
<td>0.490**</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.069)</td>
<td>(0.048)</td>
<td>(0.184)</td>
</tr>
<tr>
<td>Squared EPL index</td>
<td></td>
<td></td>
<td></td>
<td>-0.141***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.044)</td>
</tr>
<tr>
<td>Anglo-Saxon dummy</td>
<td>-0.499***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.158)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High EPL dummy</td>
<td></td>
<td></td>
<td></td>
<td>-0.663***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.127)</td>
</tr>
</tbody>
</table>

Notes: data on employment protection is from OECD (2004) and data on capital-labor ratios is from Caselli (2005). The Anglo-Saxon dummy is equal to one for Australia, Canada, Ireland, New Zealand, the UK and the USA. The high-EPL dummy is equal to one for Greece, Mexico, Portugal and Turkey. ** significant at 5% *** significant at 1%

EPL then raises the demand for capital through raising the share of senior workers. Further, the size of this effect on investment in physical capital varies with the intensity of employment protection: it is low at low levels of EPL, and higher at higher levels of EPL. This generates the hump-shape curve.

The paper is organized as follows. In Section 2, we present the benchmark model, and in Section 3, the optimal behaviour and equilibrium conditions. Section 4 presents simulation results for a large class of production functions. Section 5 extends the model to human capital and reproduces the inverted U-shape pattern. Section 6 concludes that labor market institutions sometimes have a positive role in a second-best environment.

2 The model

2.1 Overview

We base our analysis on a model of a large firm with physical capital, facing labor-market matching frictions and endogenous job destruction. Labor frictions have indeed been shown to be key in understanding the effect of employment protection on labor market flows and the demand for factors (Mortensen and Pissarides, 1999). Starting from their setup, we can explore the effect of EPL effect on the demand for capital. This requires to extend the benchmark model of EPL to endogenous capital accumulation and the so-called large firm.

However, the large firm matching model requires the derivation of a set of wage determination schedules that is more complex than the conventional Nash-bargaining solution. Indeed, the large firm, when it bargains over wages with its different workers, can exploit the possibility of complex strategic interaction à la Stole and Zwievel (1996a, 1996b). In their setup, decreasing returns to scale lead the bargaining firm (in a frictionless labor market) to raise employment above the competitive level, in order to reduce the marginal product of labor and progressively reduce wages - driven down to the reservation wage at the optimal employment level of the firm. Here, with bargaining over wage and search frictions, the same issue arises because the presence of physical capital in production imposes decreasing returns to scale in labor. Decreasing returns to scale in labor require firm to take into account that, in over-hiring, it can
reduce the marginal product of workers and therefore lead to higher profit than if the firm simply ignored these interactions. These effects were analyzed in the context of a matching model in Smith (1999), Cahuc and Wasmer (2001), Cahuc, Marque and Wasmer (2008) and Bagger et alii. (2011). Hereafter they are referred to as intrafirm bargaining.

Hence, in order to answer the question of the effect of employment protection on capital accumulation, we proceed as follows:

1. we generalize the intrafirm bargaining model to endogeneous job destruction à la Mortensen and Pissarides (1999), implying in particular to move from a countable number of categories of workers to a continuum of substitutable workers with different productivities.

2. we generalize the intrafirm bargaining model to the existence of a dual employment structure and firing taxes affecting senior workers as opposed to junior workers.

### 2.2 Production and inputs

Time is continuous and discounted at a rate $r$. We study the steady state of an economy populated by a representative firm and a unit mass of workers. The firm produces output with labor and capital in quantities $N$ and $K$ respectively. The output can either be re-invested, consumed or cover other expenses such as vacancy costs and layoff costs.

There are two sources of labor heterogeneity within the firm. First, there are two levels within the firm, workers being either junior or senior. Each status implies specificities in terms of labor regulation, wage negotiation and productivity, which we describe below. Second, workers have different levels of efficiency within the senior category (for simplicity, we assume that the entry productivity of junior workers is identical).

When hired, a worker starts as a junior and is endowed with one efficiency unit. Junior workers subsequently become senior at an exogenous rate $\lambda$. When hit by this shock, the amount of efficiency units each worker is endowed with changes too: it is equal to $\xi z$, where $\xi \geq 1$ is a parameter that reflects productivity gain from seniority, while $z$ is stochastic and drawn independently from a distribution $G$ defined over the $[0, 1]$ interval. Senior workers subsequently see the $z$ component changing at a rate $\lambda$. The new amount is drawn from the same distribution $G$. Leaving the firm implies loosing the senior hierarchy.\(^5\)

\(^4\)Elsby and Michaels (2013) study the business-cycle and cross-sectional properties of a large-firm model with endogenous job destruction. In their model, job destruction appears because of idiosyncratic shocks to firm-level productivity. In our case, it is the productivity of each individual employee that is hit stochastically. Our model is more tractable because we do not need to consider the presence of an inaction region along the labor demand schedule. The job creation and destruction relations in Mortensen and Pissarides (1994) even correspond to a specific case of the relations presented in Section 3 (when the production function is linear). However, a drawback of our model is that it does not generate zero-employment growth for a subset of firms, as Elsby and Michaels (2013) do.

\(^5\)In Mortensen and Pissarides (1994), the entry productivity is the highest. While this is a simple normalization, it yields a negative correlation between wages and tenure on the job. Because this is
The labor market is characterized by search and matching frictions. This implies the existence of frictional unemployment. In particular, the firm posts vacancies at a flow cost $c$ in order to hire workers. We denote by $V$ the mass of posted vacancies. Vacancies are filled at a rate $q(\theta)$ that depends negatively on the labor market tightness $\theta \equiv \frac{V}{1-N}$, i.e. the vacancy-unemployment ratio. This rate is derived from a matching function $m(1-N,V)$ with constant returns to scale, increasing in both its arguments, concave and satisfying the property $m(1-N,0) = m(0,V) = 0$, implying that $q(\theta) = m(\theta^{-1},1)$. Similarly, the rate at which unemployed workers find a job is equal to $\theta q(\theta)$.

The firm may endogenously choose to destroy jobs upon the revelation of productivity following an idiosyncratic shock. In particular this occurs when the stochastic component of worker’s efficiency drops below a threshold $R \in (0,1)$, the value of which is determined below. When fired, a worker comes back to the pool of unemployed. Denote by $J$ and $S \equiv \int_{1-R}^{1} s(z)dz$ the stocks of junior and senior workers respectively, where $s(z)$ is the mass of senior workers employed by the firm, who are endowed with $z$ efficiency units. Their laws of motion are described by the following equations:

$$\dot{J} = Vq(\theta) - \lambda J$$

and

$$\dot{S} = \lambda(1 - G(R))J - \lambda G(R)S.$$  (2)

The labor force is normalized to 1, and the rate of unemployment is equal to $1 - S - J = 1 - N$ where $S$, $J$ and $N = S + J$ are respectively the mass of employment of senior workers, the mass of junior workers and their sum (total employment).

We also denote by

$$Z \equiv \int_{1-R}^{1} \xi z s(z)dz$$

the mass of senior workers in efficiency units.

### 2.3 Regulation and implications for wages

We will now focus on the solutions in a stationary state in which the mass of senior workers in efficiency units $Z$ is constant. We will compute wages and in particular wages of senior workers facing productivity changes in $z$ at a fixed $Z$. When a senior worker is laid off, the firm pays a firing tax $T$. The size of this distortion influences the value of the threshold $R$. Respectively denote by $\pi_z$ and $\pi_j$ the marginal values of a senior worker endowed with $z$ efficiency units and a junior worker. The firm applies the following rule:

$$\pi_R + T = 0.$$ (3)

Wages are negotiated à la Nash between the worker and the firm. We denote by $\beta \in (0,1)$ the bargaining power of workers. Hence, the wage of a junior worker is determined as follows:

$$w_j = \arg \max \pi_j^{1-\beta} [W_j - U]^{\beta},$$ (4)

counterfactual, we introduce the $\xi$ parameter in order to obtain a positive correlation.

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6This is not due to the law of large numbers, simply a stationary assumption that the aggregate $Z$ at the firm level does not change in time.
where \( W_j \) is the present discounted value of being employed as a junior worker and \( U \) the value of unemployment. Workers earn this amount until they become senior or leave the firm.

Because the firm pays the firing tax only when a senior worker is dismissed, the firm’s threat point is different when bargaining wages with senior workers. This implies the following rule for the wage of senior workers, instead of the solution described by equation (4):

\[
w_s(z) = \arg \max \left[ \pi_z + T \right]^{1-\beta} \left[ W_s(z) - U \right]^\beta,
\]

where \( W_s(z) \) is the present discounted value of being employed as a senior worker with \( z \) efficiency units.

### 2.4 Bellman equations of workers and firms

The present discounted value of being unemployed is defined as

\[
rU = b + \theta q(\theta) [W_j - U],
\]

where \( b \) is the flow value of being unemployed, while the value of being employed as a junior worker is

\[
rW_j = w_j + \lambda \int_0^1 \left[ \max \{ W_s(z), U \} - W_j \right] dG(z)
\]

and the value of being employed as a senior worker with \( z \) efficiency units is

\[
rW_s(z) = w_s(z) + \lambda \int_0^1 \left[ \max \{ W_s(x), U \} - W_s(z) \right] dG(x).
\]

Finally the value of the firm is

\[
\Pi = \max_{V,I} \left\{ \frac{1}{1 + r dt} \left( F(J, Z, K) - w_j J - \int_R^1 W_s(z) s(z) dz - cV - I - \lambda(S + J)G(R)T \right) dt + \Pi' \right\},
\]

subject to equations (1) to (5) and

\[
\dot{K} = I - \delta K,
\]

which describes the dynamics of the capital stock. In equations (9) and (10) \( dt \) is an arbitrarily small interval of time, \( I \) is investment in capital, \( \delta \) is the capital depreciation rate, \( F \) is a constant-returns-to-scale production function that is strictly concave in each argument.

### 3 Equilibrium

#### 3.1 Aggregate employment

In the steady state, flows into aggregate employment equal flows out of employment. This leads to the following steady-state level of employment:

\[
N = \frac{\theta q(\theta)}{\theta q(\theta) + \lambda G(R)},
\]

subject to equations (1) to (5) and

\[
\dot{K} = I - \delta K,
\]

which describes the dynamics of the capital stock. In equations (9) and (10) \( dt \) is an arbitrarily small interval of time, \( I \) is investment in capital, \( \delta \) is the capital depreciation rate, \( F \) is a constant-returns-to-scale production function that is strictly concave in each argument.
while the unemployment rate is the complement to 1:

\[ u = \frac{\lambda G(R)}{\theta q(\theta) + \lambda G(R)} \]  

(12)

### 3.2 First-order conditions and factors demand

In the Appendix, we show that the first-order conditions of the factor demand problem in (9) together with the envelope theorem imply the following equilibrium conditions:

\[
(r + \lambda) \frac{c}{q(\theta)} = \frac{\partial F(J, Z, K)}{\partial J} - w_j J - \int R \frac{\partial w_s(z)}{\partial J} s(z) dz - \lambda T \int R [\pi z + T] dG(z),
\]  

(13)

\[
(r + \lambda) (\pi z + T) = \frac{\partial F(J, Z, K)}{\partial Z} J - \int R \frac{\partial w_s(z)}{\partial J} s(z) dz + r T + \lambda \int R [\pi x + T] dG(x)
\]  

(14)

and

\[ r + \delta = \frac{\partial F(J, Z, K)}{\partial K} J - \int R \frac{\partial w_s(x)}{\partial K} s(x) dx. \]  

(15)

Equation (13) describes the incentives for the firm to open up new vacancies. The left-hand side represents the marginal cost of filling a junior vacancy, while the right-hand side is the marginal revenue the marginal junior worker brings to the firm—equal to \((r + \lambda)\pi_j\), that is, its marginal product (first term) net of the wage (second term) and the effect of this hiring on the wage of the other workers (third and fourth terms). The marginal revenue also takes into account the possibility that the marginal junior worker may become a senior at a rate \(\lambda\). In this case, a firing tax may be paid (fifth term) and the surplus for the firm changes to a new value (last term).

Equation (14) can be understood in a similar way, with the difference that \(T\) positively affects the firm surplus in this case. This is the standard effect that explains why \(T\) has an ambiguous impact on unemployment in Mortensen and Pissarides (1999): once workers become senior, the firm’s threat point drops because the firing tax \(T\) has to be paid in case of a layoff.

Finally, (15) characterizes the firm’s capital investment decision. The firm purchases capital such that the opportunity cost of capital (the left-hand side) equalizes the marginal revenue (the right-hand side). The latter is composed by the marginal product of capital and the effect of the capital stock on wages.

### 3.3 Strategic bargaining

We have previously illustrated that the employment and capital stocks affect wages. Both workers and firms take this into account when negotiating wages. In particular, firms use hiring decisions as a way to strategically affect the marginal product of labor and in turn, on equilibrium wages. This is the logic of intrafirm bargaining in Stole and Zwiebel (1996a and b), extended to search models in Smith (1999), Cahuc and Wasmer (2001), Cahuc, Marque and Wasmer (2008) and Bagger and alii (2011). In the Appendix, we show that the solutions for wages resulting from these strategic interactions follow a simple rule. The wage is a weighted average of
the reservation value of the worker $rU$ and of the marginal product of each type of labor, augmented (for senior workers) or diminished (for junior workers) of the value of layoff costs:

$$w_j = (1 - \beta)rU - \beta \lambda T + \beta \Omega_j \frac{\partial F(J, Z, K)}{\partial J}$$

(16)

in the case of the junior wage and

$$w_s(z) = (1 - \beta)rU + \beta rT + \beta \Omega_s z \xi \frac{\partial F(J, Z, K)}{\partial Z}$$

(17)

for senior, where

$$\Omega_j = \int_0^1 \frac{1 - \beta}{\beta} \frac{\partial F(Jx, Zx, K)}{\partial J} dx$$

(18)

and

$$\Omega_s = \int_0^1 \frac{1 - \beta}{\beta} \frac{\partial F(Jx, Zx, K)}{\partial Z} dx$$

(19)

are over-employment factors resulting from strategic interactions. They are derived and discussed in Cahuc et alii. (2008, Section 2): when $\Omega_j$ and $\Omega_s$ are equal to one, the solution is the one described in Mortensen and Pissarides (1999) when the production technology is such that returns to labor are constant. When they differ from one, e.g. with decreasing returns to scale in labor, we are away from Mortensen and Pissarides’ solution: they describe a situation of “over-employment” if they take a value larger than 1 and “underemployment” if their value is below 1.

3.4 Job creation and destruction

By replacing the solution for wages in (13) and (14), we obtain the following job creation rule:

$$\frac{c}{q(\theta)} = (1 - \beta) \left[ \Omega_j \frac{\partial F(J, Z, K)}{J} - \xi R \Omega_s \frac{\partial F(J, Z, K)}{Z} - T \right]$$

(20)

In the Appendix, we also show that job destruction rule (3) can be rewritten as follows:

$$0 = \xi \Omega_s \frac{\partial F(J, Z, K)}{Z} \left( R + \frac{\lambda}{r + \lambda} \int_R^1 (z - R) dG(z) \right) - b - \frac{\beta}{1 - \beta} c \theta + rT.$$  

(21)

Again, these conditions differ from Mortensen and Pissarides’ through the presence of the over-employment factors and the fact that marginal products of labor are not necessarily constant.

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7Note here a technical issue that do not arise in the previous works cited above: these wage solutions here are obtained under the assumption that each productivity type $z$ of senior workers can be analyzed as a representative senior worker who bargains with the firm. This is possible because agents in this large firm model are perfect substitutes in production.
3.5 Capital demand

Using the wage equations (16) and (17), we can rewrite the capital demand as

\[(1 - \beta)\Omega_k \frac{\partial F (J, Z, K)}{\partial K} = r + \delta,\]  \hspace{1cm} (22)

where

\[\Omega_k = \int_0^1 \frac{1}{\beta} x^{1-\beta} \left( \frac{\partial F (J, Z, K)}{\partial K} \right) dx\]  \hspace{1cm} (23)

is an over-investment factor that takes the condition away from the neo-classical investment model when its value differs from one, and is identical to the expression in Cahuc et alii. (2008, Section 4).

4 Numerical examples

We resort to numerical examples to illustrate the various effects of employment protection, reflected by the tax on lay-offs \(T\) in our model. Our strategy is as follows: we first find a set of parameters that approximate an economy with labor-market characteristics similar to the United States. This will serve our purpose which is to decompose the effects of EPL on capital-labor ratios along its different dimensions.

In this economy, taxes on lay-offs are absent. We then ask how macroeconomic aggregates in this economy are affected by the introduction of a firing tax. Our examples are consistent with the well-known results in the literature, such as the ambiguous impact on employment: we will obtain a positive effect on job tenure and unemployment duration and a lower level of labor turnover. We will also find that, in most specifications, employment protection lowers the incentives to accumulate capital, and negatively affects capital-to-labor ratios.

4.1 Numerical exercise

Our benchmark economy resembles pretty well the economy described in Pissarides (2009). The reason is because the theoretical model in Pissarides (2009) is a particular case of the theoretical model we describe in Section 2. It corresponds to the case where the distribution \(G\) is degenerate in zero, the firing tax \(T\) is null and the production function is linear in each of its arguments, which impedes the firm from over-hiring. Hence, we borrow a great deal from Pissarides (2009) and share many of his parameter values.\(^8\)

We assume that a unit interval of time corresponds to a month. We set the discount rate at a 4% annual rate. The matching function is assumed to be Cobb-Douglas \(m(1 - N, V) = m_0(1 - N)^\eta V^{1-\eta}\), with unemployment elasticity \(\eta = 0.5\). We also follow common practice by setting \(\beta = \eta\). This internalizes the search externalities in the standard one-worker-per-firm model.\(^9\) We follow Pissarides by targeting a

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\(^8\)See also Janiak (2010).

\(^9\)Notice that the Hosios-Pissarides efficiency rule only applies to specific cases of the model of Section 2. When the firm chooses to over-employ, the rule has to be modified in order to account for this additional externality. See Smith (1999).
<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
<th>Parameter</th>
<th>Target/Source</th>
</tr>
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<tbody>
<tr>
<td>$r$</td>
<td>0.0033</td>
<td>Discount rate</td>
<td>4% annual rate</td>
</tr>
<tr>
<td>$b$</td>
<td>0.71</td>
<td>Flow value of unemployment</td>
<td>Hall and Milgrom (2008)</td>
</tr>
<tr>
<td>$c$</td>
<td>0.356</td>
<td>Vacancy cost</td>
<td>Hall and Milgrom (2008)</td>
</tr>
<tr>
<td>$m_0$</td>
<td>0.7</td>
<td>Scale parameter (matching)</td>
<td>vacancy-unemployment ratio</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.5</td>
<td>Elasticity (matching)</td>
<td>Pissarides (2009)</td>
</tr>
<tr>
<td>$\beta$</td>
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<td>Bargaining power</td>
<td>$\beta = \eta$</td>
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<td>$\lambda$</td>
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<td>Productivity shock frequency</td>
<td>job separation rate</td>
</tr>
<tr>
<td>$A$</td>
<td>0.4372</td>
<td>Total factor productivity</td>
<td>job finding rate</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.4309</td>
<td>Return to seniority</td>
<td>6% annual rate</td>
</tr>
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<td>Labor share</td>
<td>standard</td>
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<tr>
<td>$\delta$</td>
<td>0.0087</td>
<td>Capital depreciation rate</td>
<td>10% annual rate</td>
</tr>
</tbody>
</table>

The parameters $b = 0.71$ and $c = 0.356$, which respectively correspond to the flow value of being unemployed and the flow cost of keeping a vacancy posted, are also taken from Pissarides (2009). Those values are consistent with two facts from Hall and Milgrom (2008). First, the flow value of nonwork produces a realistic gap between the flow utility of employment and unemployment. Second, the value of $c$ generates recruiting costs equal to 14 percent of quarterly pay per hire, which is in line with evidence reported in Silva and Toledo (2009).
Finally the capital depreciation rate $\delta$ remains to be identified. We assume a 10% annual depreciation rate. This is consistent with evidence in Gomme and Rupert (2007), who report depreciation rates for different sorts of capital. Ten percent corresponds to the annual rate averaged across all market types of capital in their paper. This produces a share of investment in aggregate output equal to 14.6%.

4.2 Benchmark simulations

The comparative statics exercise in this subsection is close to Ljungqvist’s (2002). We compute steady-state equilibria for different values of the firing tax $T$: all parameters in the model are as in the benchmark economy but the tax on layoffs, and we consider varying values of $T$, that range from 0 to 10. The latter value corresponds to a tax approximately equal to one year of an average worker’s salary.

4.2.1 Labor market effects

Figure 2 confirms earlier findings in the literature on the impact of employment protection in the labor market. Given that these results are well-undestood, we will only briefly describe them. First, it is easy to understand that the introduction of a
firing tax lowers labor turnover.\textsuperscript{10} Second, and given this the effect on unemployment is ambiguous. Unemployment increases if incidence (job flows into unemployment) falls by proportionnaly less than duration increases.\textsuperscript{11} For the simulation we propose, it turns out that the firing tax has a positive impact on the rate of unemployment. For a tax as large as a year of an average worker’s wage, the associated unemployment rate is about 7.5\% (as opposed to 6\% in the benchmark economy). We illustrate in Section 4.3 that the unemployment rate may decrease under another parametrization.

\textbf{4.2.2 Results on the accumulation of capital}

Figure 3 presents results for the use of capital. Beyond labor variables such as total employment, labor productivity and the share of young workers, we report: i) the effect on the aggregate stock of capital (first panel), ii) the capital-labor ratio (third panel) and iii) the ratio of capital to the whole stock of efficiency units in the firm (sixth panel). The Figure shows that the first two elements fall when the size of the firing tax increases, while the third one is not affected by employment protection.

The intuition for these results is the following. First, we already know from Figure 2 that the employment level drops when a firing tax is introduced. Hence, the second panel in Figure 3 shows the same information as the fourth panel in Figure 2.

Going back to the capital demand equation (22), and with the particular structure imposed by our calibration strategy on the production function, the number of arguments that appear in the marginal product of capital in that equation reduces to two: the total stock of efficiency units of labor \((J + Z)\) and capital \(K\). Moreover the over-hiring factor \(\Omega_k\) becomes a constant equal to \((1 - \beta + \beta \alpha)^{-1}\). Hence, it suffices to follow the total stock of efficiency units of labor: capital adjusts such that its marginal income (the left-hand side of (22)) equals its opportunity cost (the right-hand side).

Precisely, in our simulations, a firing tax decreases the total stock of efficiency units for three reasons. First, employment falls, as in the second panel and as already discussed in Section 4.2.1. Second, the drop in the reservation productivity \(R\) makes senior workers less productive on average. This last effect has been largely studied in the literature, as in Hopenhayn and Rogerson (1993), Mortensen and Pissarides (1999), Veracierto (2001) and Lagos (2006). Third, the composition of employment is affected: as the reservation productivity \(R\) decreases the share of junior workers in employment decreases (see fourth panel).\textsuperscript{12}

\textsuperscript{10}As shown on the third panel of the Figure, a higher firing tax is associated with a lower job separation rate. Because the firm is reluctant to pay the tax, it is willing to keep some low-productivity workers, which would have been dismissed absent labor regulation. This implies a lower threshold \(R\) (see the first panel) as condition (3) suggests that the marginal value of the least productive worker becomes more negative at larger \(T\). As a result, the job separation rate is also lower. In turn, employment protection generates lower labor-market tightness and job finding rate (see the first and second panel respectively): because the firm anticipates to pay the firing tax at some moment once a worker is hired, incentives to open up new vacancies are reduced \textit{ex ante}. This negatively affects the probability to find a job for an unemployed worker, which is confirmed by the job creation condition (20): for given values of \(R\) and \(K\), \(\theta\) is lower at higher \(T\).

\textsuperscript{11}See Mortensen and Pissarides (1999).

\textsuperscript{12}This comes from the standard Mortensen-Pissarides (1994) assumption of a higher productivity for junior workers than senior workers: this composition effect negatively impacts the total stock of efficiency units.
Since the total stock of efficiency units in the economy decreases, the aggregate stock of capital has to decrease too in order to keep the marginal income of capital equal to its opportunity cost. This is a direct consequence of the constant-returns-to-scale nature of the production function, which implies that labor efficiency units and capital are complements in the production function. The first panel accordingly shows a drop in the aggregate stock of capital following an increase in the firing tax. As an illustration, our simulation suggests that the introduction of a tax equal to a year of an average worker’s wage in the benchmark economy implies a fall by 15% in the capital stock. This fall completely reflects the drop in the stock of labor efficiency units.

It is easier to understand the fall in the capital-labor ratio by first understanding the effect on the ratio of capital to the whole stock of efficiency units. Because the production function is homogeneous of degree one, its derivative (the marginal product) is homogeneous of degree zero. This implies that the ratio of capital to the stock of labor efficiency units $\hat{k} = K/(J + Z)$ is constant and only determined by the opportunity cost of capital. Thus, $\hat{k}$ has to be independent of employment protection for the particular production function assumed in the calibration exercise, as observed on the last panel of Figure 3.
The observations on the first and last panels of the Figure consequently help the analysis of the third panel, which displays the impact on the capital-labor ratio. Since $K$ decreases and $\tilde{k}$ remains constant, the ratio of $K$ to $N$ has to decrease. As an illustration, the Figure shows that the introduction of a tax equal to a year of an average worker’s wage generates a decrease by 13% in the ratio. This decrease has to occur to compensate the decrease in the average productivity of workers of a similar size.

A simple summary of the different mechanisms is as follows:

- Employment protection $T\uparrow \Rightarrow$ Number of senior workers $S\uparrow$, Number of junior workers $J\downarrow$

- Employment protection $T\uparrow \Rightarrow$ Average efficiency of senior workers $Z/S \downarrow$, Average efficiency of labor $(J+Z)/N \downarrow$,

- Capital per efficiency unit of labor $K/(J+Z)$ remains constant but capital per unit of labor $k \downarrow$.

4.3 Alternative production technologies

In this subsection, we discuss the robustness of the results displayed on Figures 2 and 3. We consider four alternative parametrizations, for which Figure 4 shows the comparative statics: it reports the effect of a firing tax on employment, capital, the capital-labor ratio and capital per efficiency units, the latter being defined as $\tilde{k} \equiv \frac{K}{J+Z}$.

In the first parametrization, which we label “low matching efficiency”, we illustrate that the impact of the firing tax on employment can be positive. As emphasized by Ljungqvist (2002), in matching models with highly frictional labor markets, lay-off costs tend to increase employment by reducing labor reallocation. This parametrization accordingly considers an economy with the same production function as in the benchmark economy, but it enhances the degree of search frictions in the labor market. This is done by setting the scale parameter in the matching function equal to half its value in the benchmark economy. Figure 4 confirms this idea: the dashed line shows that employment is increasing in $T$. The effects on capital are qualitatively similar as in Figure 3, for the same reasons given in Section 4.2.2.

The second alternative parametrization, labelled “Cobb-Douglas J-Z-K”, considers another Cobb-Douglas production function of the form $F(J,Z,K) = AJ^{\alpha}Z^{\gamma}K^{1-\alpha-\gamma}$. It shows that, though the effect of the firing tax on capital and capital per worker is negative, it may be positive for the stock of capital per efficiency units.

Third, we consider a situation where the impact on both the capital stock and the capital-labor ratio is positively affected by an increase in the firing tax. We label this parametrization as “J and Z-K additively separable”. We consider a production function of the form $F(J,Z,K) = A [J + \gamma Z^{\alpha}K^{1-\alpha}]$, which implies that capital and efficiency units provided by senior workers are complements in production as in the benchmark specification, but junior workers do not affect the marginal product of

\footnote{In all the alternative parametrizations, we choose the values of the TFP $A$ and the arrival rate $\lambda$ by targeting a job finding rate of 59.4% and a job separation rate of 3.6%. The parameter $\xi$ is simply fixed to 1.}
Figure 4: Capital and capital-to-labor ratio as a function of the firing tax $T$, alternative parametrizations

Note: capital, employment, capital-to-labor ratio and the ratio of capital to efficiency units are all normalized to one for a $T$ value of zero.

capital anymore. Hence, because an increase in the firing tax generates an increase in the share of senior workers in employment, the firm is given incentives to invest in capital.

Finally, we present the case where separations are exogenous. In this situation, the production function only depends on two factors, i.e. labor and capital. As a consequence, the capital-labor ratio is constant and independent of $T$. The reason for this is the same reason why the stock of capital per efficiency units is independent of $T$ in Figure 3: the marginal product of capital is homogeneous of degree zero. Moreover, the aggregate level of employment decreases as in Pissarides (2000) and the capital stock is negatively affected because of its complementarity with labor in the production function. In the Appendix A.2, we formally show these comparative statics. We also study the case with decreasing returns to scale, where the capital-labor ratio is increasing in $T$. 
5 Extension: Human capital and the U-shape pattern

Insofar, the model has failed to obtain a positive link between EPL and the capital-labor ratio except for a specific production technology where senior workers are complements with physical capital. Further, none of these production functions replicate a hump-shaped pattern.

We will show here that an extension of the model with search frictions where senior workers invest in specific skills, is precisely able to reproduce this inverted U-shape pattern. We add the following ingredient: workers now have the possibility to invest in specific skills at the time of entry. They do so at cost $C(h)$ on the spot, with $C'(h) > 0$ and $C''(h) \geq 0$. Human capital adds up to productivity when workers become senior. The $\xi$ component is thus made endogenous in this version of the model.\footnote{See also Wasmer (2006) for a model with search frictions, firing taxes and investment in specific skills but no intrafirm bargaining and no physical capital investment.}

5.1 Human capital

When hired, a worker still starts as a junior and is endowed with one efficiency unit. Junior workers then become senior at an exogenous rate $\lambda$ where its productivity is $h,z$ and $z$ is drawn from a distribution $G$ defined over the $[0,1]$ interval. Their amount of efficiency units still changes at a rate $\lambda$ in the same distribution $G$. Leaving the firm implies now both the loss of specific skills and seniority.

The present discounted value of being unemployed is defined as

$$rU = b + \theta q(\theta) \left[ W_j(h^*) - C(h^*) - U \right],$$

(24)

where $h^*$ is the optimal stock of human capital, while the value of being employed as a junior worker is

$$rW_j(h) = w_j(h) + \lambda \int_0^1 \left[ \max \{ W_s(h,z), U \} - W_j(h) \right] dG(z)$$

(25)

and the value of being employed as a senior worker with $z$ efficiency units is

$$rW_s(h,z) = w_s(h,z) + \lambda \int_0^1 \left[ \max \{ W_s(h,x), U \} - W_s(h,z) \right] dG(x).$$

(26)

Finally the value of the firm is

$$\Pi = \max_{V,I} \frac{1}{1 + rdT} \left\{ \left( F(J, Zh, K) - w_j(h)J - \int_{R(h)}^1 w_s(h,z) s(z) dz \right. \right.$$  

$$\left. - cV - I - \lambda(S+J)G(R(h))T \right) dt + \Pi' \}$$

(27)

subject to equations (1) to (5) and (10).

The first-order condition for human-capital investment reads as

$$(r + \lambda)C'(h) = \frac{dw_j(h)}{dh} + \frac{\lambda}{r + \lambda G(R)} \int_R^1 \frac{dw_s(h,z)}{dh} dG(z).$$

(28)
It is straightforward to see from the equation above that, for a given marginal effect of $h$ on wages and a convex function $C$, lower $R$ implies higher investment $h$. The reason is because when $R$ is low, workers anticipate longer tenure on the job, which increases the marginal return on human capital.

Given that investment in human capital can only be made upon entry, the marginal cost of human capital investment is not considered in Nash bargaining: it is sunk. Assume the firm takes $h$ as given. Then, given the results from Section 3, it is easy to show that the job creation and destruction rules respectively write as

$$\frac{c}{q(\theta)} = (1 - \beta) \left[ \frac{\Omega_j \frac{\partial F(j,hZ,K)}{\partial J} - hR \Omega_s \frac{\partial F(j,hZ,K)}{\partial (hZ)}}{r + \lambda} - T \right]$$

(29)

and

$$0 = h \Omega_s \frac{\partial F(j,hZ,K)}{\partial (hZ)} \left( R + \frac{\lambda}{r + \lambda} \int_R^1 (z - R)dG(z) \right) - b - \frac{\beta}{1 - \beta} c\theta + rT$$

(30)

in this framework. Notice that $h$ now appears as a multiplicative term next to $R$ in the conditions (29) and (30).

Additionally, the demand for capital is given by

$$(1 - \beta) \Omega_k \frac{\partial F(j,hZ,K)}{\partial K} = r + \delta$$

(31)

and the equation for the aggregate level of employment (11) remains the same.

Finally, we show in the Appendix that condition (28), which describes the incentives of supplying human capital, can be rewritten as

$$(r + \lambda)C'(h) = \frac{\lambda}{r + \lambda G(R)} \beta \Omega_s \frac{\partial F(j,hZ,K)}{\partial (hZ)} \int_R^1 z dG(z)$$

(32)

once wages are replaced by their equilibrium values.

5.2 Numerical illustration

We proceed as in Section 4.2 and illustrate the effect of a firing tax on capital through numerical simulations. The addition of human capital generates new mechanisms, involving in particular the ability of workers to respond to incentives in changing their investment in human capital. When human capital changes, so does the demand for capital. The ability to vary the level of human capital depends in turn on the cost of investing. A flexible parametrization of the cost function is

$$C(h) = \sigma_0 + \sigma_1 h^{\sigma_2}.$$ 

(33)

The parameter $\sigma_0$ is introduced so that the participation constraint $[W_j(h^*) - U] \geq C(h^*)$ is satisfied, $\sigma_1$ is a scale parameter and $\sigma_2 \geq 1$ influences the elasticity of the supply function of human capital. We also choose the same production function as in our benchmark economy,

$$F(J,hZ,K) = A (J + hZ)^{\alpha} K^{1-\alpha}.$$
Since this extension of the model simply endogenizes the $\xi$ component in Section 2, we can borrow a great deal from the calibration strategy in Section 4.1. In fact, we can keep most of the parameters of Table 2 and choose the parameters of the cost function (33) so as to target an equilibrium value for $h$ equal the parameter $\xi$ of Section 4.1: this allows us to obtain the same returns to seniority as in Section 4.1. Now, there are many combinations of the parameters $\sigma_1$ and $\sigma_2$ that lead a 6% annual return. Our strategy is to fix $\sigma_2$ to a specific value and then obtain the value of $\sigma_1$ that produces a 6% annual rate of return to seniority. We choose $\sigma_2 = 7.5$, giving a supply elasticity of human capital equal to 0.13. The resulting value for $\sigma_1$ is 0.22. In the Appendix, we show how the comparative statics of the effect of $T$ change when we consider different elasticities for the supply of human capital.

Figure 5 illustrates the hump-shaped effect of employment protection. The Figure displays the steady-state values of capital, employment, capital-labor ratio, human capital, tightness and reservation productivity for each one of the cases respectively. To understand the effect on capital accumulation, remember that capital is a complement of effective workers for the production function of the benchmark economy. When the stock of effective workers increases, the stock of physical capital has to increase too and it it declines when a decrease in the stock of effective workers occurs. A firing tax thus has two opposite consequences on the demand for capital: the first one, as in Mortensen and Pissarides (1994), is to reduce the reservation threshold of idiosyncratic productivity $R$ and therefore its average level, and finally the demand for physical capital; the second one is to reduce labor turnover and thus raise the incentives to invest in human capital, at a constant level of the physical capital. This can be seen from the job creation and destruction conditions (29), (30) and the human capital supply function (32). The increasing part of panels (1) and (3) in Figure 5 is thus due to the fact that the second effect dominates the first one, while, along the decreasing part of the curves, it is the first effect that dominates.

6 Conclusions

This paper has attempted to clarify the role of a specific employment regulation, employment protection, on economic outcomes such as capital and capital-labor ratio. We have shown that the main effect of employment protection on investment and the capital-labor ratio is a quite robust negative one: because EPL reduces future profits, firms underinvest in physical capital. This is a variant of the hold-up effect. We view this set of results as short-run ones.

However, by protecting skills, employment protection can also increase the investment in physical capital and therefore, under the realistic assumption of complementarity between physical and human capital, the effect of employment protection can be mitigated and even reversed.

This points out that labor institutions have ambiguous effects: sometimes negative, but in specific contexts sometimes positive. It is a second best result arising from the presence of contractibility issues over the protection of investment in physical and human capital (hold-up). It however suggests that, along the line of the new public economics literature, that future research should more systematically investigate the positive role of labor market institutions, and in particular enrich empirical specifications in order to account for such impacts on long-run efficiency of labor as well as
Figure 5: The impact of a firing tax ($T$) in the model with specific human capital

Note: physical capital, employment, the ratio of physical capital to labor, and human capital are all normalized to one for a $T$ value of zero.

short-run efficiency costs.

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A Appendix

A.1 Equilibrium

A.1.1 First-order conditions

Differentiating the right hand side of (9) with respect to $V$ and setting it equal to zero gives

$$-c + \pi_j q(\theta) = 0,$$

while the condition for $I$ is

$$\pi_k = 1,$$

with $\pi_k$ being the marginal value of capital.

Application of the envelope theorem yields (14),

$$(r + \lambda)\pi_j = \partial F(J, Z, K) \over \partial J - \int_R^1 \partial w_s(x) \over \partial J s(x) dx - \lambda T + \lambda \int_R^1 [\pi_x + T] dG(x),$$

and

$$(r + \delta)\pi_k = \partial F(J, Z, K) \over \partial K - \int_R^1 \partial w_j \over \partial K J - \int_R^1 \partial w_s(x) \over \partial K s(x) dx.$$ (37)

Conditions (13) and (15) are obtained by combining (34) with (36) and (35) with (37) respectively.

A.1.2 Wages

The Nash bargaining rule (4), combined with (36) and (7), give the following formulation for wages of junior workers:

$$w_j = \beta \left[ \partial F(J, Z, K) \over \partial J - \int_R^1 \partial w_j \over \partial J J - \int_R^1 \partial w_s(x) \over \partial J s(x) dx - \lambda T + \lambda \int_R^1 [\pi_x + T] dG(x) \right]$$

$$- (1 - \beta) \left[ \lambda \int_R^1 [W_s(z) - U] dG(z) - rU \right].$$ (38)

The same Nash bargaining rule allows to establish that

$$(1 - \beta)\lambda \int_R^1 [W_s(z) - U] dG(z) = \beta \lambda \int_R^1 [\pi_x + T] dG(x).$$ (39)

Hence,

$$w_j = \beta \left[ \partial F(J, Z, K) \over \partial J - \int_R^1 \partial w_j \over \partial J J - \int_R^1 \partial w_s(x) \over \partial J s(x) dx - \lambda T \right] + (1 - \beta) rU.$$ (40)

Similarly, the rule (5) together with (14) and (8), imply that

$$w_s(z) = \beta \left[ \partial F(J, Z, K) \over \partial Z z \xi - \int_R^1 \partial w_j \over \partial s(z) J - \int_R^1 \partial w_s(x) \over \partial s(z) s(x) dx + rT \right] + (1 - \beta) rU.$$ (41)

Given the results by Cahuc et al. (2008), we conjecture that the solution to the system of differential equations described by (40) and (41) is (16) and (17). To verify our
conjecture in the case of the wage of junior workers, we derive (16) and (17) with respect to $J$. We obtain

$$\frac{\partial w_j}{\partial J} = \int_0^1 x^{\frac{1}{2}} \frac{\partial^2 F(Jx, Zx, K)}{\partial (Jx) \partial (Jx)} dx$$

(42)

and

$$\frac{\partial w_s(z)}{\partial J} = \int_0^1 x^{\frac{1}{2}} \frac{\partial^2 F(Jx, Zx, K)}{\partial (Zx) \partial (Jx)} z dx.$$  

(43)

Our conjecture is correct if

$$\hat{1}0 x \frac{\partial}{\partial (Jx)} \frac{\partial F(Jx, Zx, K)}{\partial (Jx)} dx = \beta \frac{\partial}{\partial J} (1 - \beta) b + \beta \theta c - \beta \lambda T$$

(44)

Replacing the derivatives of (16) and (17) with respect to $J$ in the equation above and integrating by parts reveals that the conjecture is correct.

The conjecture can be verified in a similar way in the case of wages of senior workers.

Moreover, notice that (16) and (17) can be rewritten as

$$w_j = \beta \Omega_j \frac{\partial F(J, Z, K)}{\partial J} + (1 - \beta) b + \beta \theta c - \beta \lambda T$$

(45)

and

$$w_s(z) = \beta \Omega_s z \xi \frac{\partial F(J, Z, K)}{\partial Z} + (1 - \beta) b + \beta \theta c + \beta r T,$$

(46)

by use of equation (6) together with (4) and (34).

**A.1.3 Job destruction**

The job destruction relation is obtained as follows. First, replace wages and their derivatives in (36) to get

$$(r + \lambda) \pi_j = (1 - \beta) \left[ \Omega_j \frac{\partial F(J, Z, K)}{\partial J} - rU - \lambda T \right] + \lambda \int_R^1 (\pi_z + T) dG(z).$$

(47)

Similarly, from (14), we have

$$(r + \lambda) (\pi_z + T) = (1 - \beta) \left[ \Omega_s \frac{\partial F(J, Z, K)}{\partial Z} z \xi - rU + r T \right] + \lambda \int_R^1 (\pi_z + T) dG(z).$$

(48)

It follows that

$$(r + \lambda) (\pi_z - \pi_R) = (1 - \beta) \Omega_s \frac{\partial F(J, Z, K)}{\partial Z} (z - R) \xi.$$ 

(49)

Given that $\pi_R + T = 0$,

$$\pi_z = \frac{1 - \beta}{r + \lambda} \Omega_s \frac{\partial F(J, Z, K)}{\partial Z} (z - R) \xi - T.$$  

(50)

Evaluating (48) for $z = R$ and combining it with (50) yields

$$0 = (1 - \beta) \left[ \Omega_s \frac{\partial F(J, Z, K)}{\partial Z} R \xi - rU + r T \right] + \lambda \int_R^1 \frac{1 - \beta}{r + \lambda} \Omega_s \frac{\partial F(J, Z, K)}{\partial Z} \xi (z - R) dG(z).$$

(51)
Finally, replacing $rU$, in the equation above, by

$$rU = b + \theta \frac{\beta}{1 - \beta} c$$  \hfill (52)

gives (21).

### A.1.4 Job creation

Combining (36) with (14), together with the solution for wages, we have

$$(r + \lambda)(\pi_j - \pi_z) = (1 - \beta) \left( \Omega_j \frac{\partial F(J, Z, K)}{\partial J} - \Omega_s \frac{\partial F(J, Z, K)}{\partial Z} z\xi \right) + \beta(r + \lambda)T. \hfill (53)$$

If we notice that $\pi_R + T = 0$ and given the result in (34), we can obtain equation (20) by evaluating this equation at $z = R$.

### A.1.5 Capital

The capital demand (22) is obtained from the marginal value of capital (37). Notice first that the derivatives of wages with respect to capital write as

$$\frac{\partial w_j}{\partial K} = \int_0^1 x^{\frac{1 - \beta}{\beta}} \frac{\partial^2 F(J, Z, K)}{\partial (J, z\xi) \partial K} dx$$  \hfill (54)

and

$$\frac{\partial w_s(z)}{\partial K} = \int_0^1 x^{\frac{1 - \beta}{\beta}} \frac{\partial^2 F(J, Z, K)}{\partial (Z, z\xi) \partial K} z\xi dx. \hfill (55)$$

Replacing them into (37) implies

$$r + \delta = \frac{\partial F(J, Z, K)}{\partial K} - \int_0^1 Jx \frac{1 - \beta}{\beta} \frac{\partial^2 F(J, Z, K)}{\partial (J, z\xi) \partial K} dx - \int_0^1 Zx \frac{1 - \beta}{\beta} \frac{\partial^2 F(J, Z, K)}{\partial (Z, z\xi) \partial K} dx,$$  \hfill (56)

which is equivalent to

$$r + \delta = \frac{\partial F(J, Z, K)}{\partial K} - \int_0^1 x^{\frac{1 - \beta}{\beta}} \left[ J \frac{\partial^2 F(J, Z, K)}{\partial (J, z\xi) \partial K} + Z \frac{\partial^2 F(J, Z, K)}{\partial (Z, z\xi) \partial K} \right] dx. \hfill (57)$$

By integrating by parts the integral in the equation above, one can get equation (22).

### A.2 The model with exogenous separations

In this section we analyze a slightly different version of the model, with exogenous job separations. We show that an increase in the firing tax on the capital-labor ratio is positive or null depending on the form one assumes for the production function.

#### A.2.1 Equilibrium

Instead of assuming that the amount of efficiency units endowed by workers evolves stochastically according to a Poisson process, we consider that all workers own the same unit amount. This implies that the firm does not choose to destroy jobs endogenously as in the main text and labor heterogeneity does not appear anymore. Now separations occur exogenously at a rate $s$. 

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In this setting the relations describing the equilibrium of the economy are the following:

\[ N = \frac{\theta q(\theta)}{q(\theta)} + s, \]  

\[ \frac{c}{q(\theta)} = \frac{(1 - \beta) \left( \Omega_n \frac{\partial F(N,K)}{\partial N} - b - sT \right) - \beta \theta c}{r + s}, \]  

and

\[ (1 - \beta) \Omega_k \frac{\partial F(N,K)}{\partial K} = r + \delta, \]  

where

\[ \Omega_n = \int_0^1 \frac{1}{x} \frac{\partial F(N_x,K)}{\partial N} dx \]  

and

\[ \Omega_k = \int_0^1 \frac{1 - \beta}{x} \frac{1 - \beta}{x} \frac{\partial F(N_x,K)}{\partial K} dx. \]

Equations (58) and (60) are the counterparts of (11) and (22) when separations are exogenous and (59) is the equivalent of the free-entry condition from Pissarides (1985).

Notice that we consider that workers earn the outside wage as in Pissarides (2000).

**A.2.2 The impact of a firing tax on the capital-labor ratio**

With exogenous separations, the effect of a firing tax on the capital-labor ratio is analogous to the effect of a labor tax in a frictionless framework. Two effects appear. First, firms substitute away from labor because the relative cost of labor increases. This can be shown by manipulating equations (59) and (60), which leads to

\[ \frac{(1 - \beta) \Omega_k \frac{\partial F(N,K)}{\partial K}}{\Omega_n \frac{\partial F(N,K)}{\partial N}} = \frac{r + \delta}{(r + s) \frac{c}{q(\theta)} + \beta \theta c + (1 - \beta)(b + sT)}. \]

The left hand side of the equation is analogous to the standard marginal rate of transformation that appears in relations describing the equilibrium of a walrasian economy, while the right hand side is analogous to the relative cost of labor. We see from this equation that an increase in \( T \) leads to an increase in the capital-labor ratio, for a given labor-market tightness \( \theta \).

Of course, the labor-market tightness also reacts to a change in \( T \). This observation leads us to a second effect: an increase in \( T \) implies a decrease in \( \theta \). This effect goes in the opposite direction as it negatively affects the capital-labor ratio.

We now illustrate those two effects through two specific examples: i) the case where the production function displays constant returns to scale and ii) a Cobb-Douglas case with decreasing returns.

Let us first consider the case with constant returns to scale. Under this assumption, it is easy to see from equation (60) that the capital-labor ratio is not affected by an increase in the firing tax \( T \). Moreover, given this result, the labor-market tightness has to decrease (see equation (59)) as well as employment (see equation (58)). Because the capital-labor ratio is not affected and \( N \) decreases, the aggregate stock of capital has to decrease too.
In the second example, we assume the production function takes the following form:

$$F(N, K) = N^\alpha K^\nu,$$

with $\alpha > 0$, $\nu > 0$ and $\alpha + \nu < 1$.

With this production function, equations (59) and (60) can be rewritten as

$$\frac{c}{q(\theta)} = \frac{(1 - \beta) \left( \frac{a k^\nu N(\theta)^{\alpha + \nu - 1}}{1 - \beta + \beta \alpha} - b - sT \right) - \beta \theta c}{r + s}$$

and

$$\frac{1 - \beta}{1 - \beta + \beta \alpha} N(\theta)^{\alpha + \nu - 1} k^\nu = r + \delta$$

respectively, where $k \equiv K N$ and $N(\theta)$ is given by equation (58), an increasing relation of $\theta$.

Equation (65) describes an increasing relation between the capital-labor ratio $k$ and the labor-market tightness $\theta$, while equation (66) is decreasing in the space $(k, \theta)$. Hence, an increase in $T$ leads to a decrease in $\theta$ and an increase in $k$ with this production function.

### A.3 The model with human capital

#### A.3.1 Equilibrium

The worker’s maximization program is given by

$$\max_h W_j(h) - C(h),$$

with the participation constraint

$$W_j(h) - U \geq C(h).$$

This leads to the first-order condition:

$$(r + \lambda)C'(h) = \frac{dw_j(h)}{dh} + \lambda \int_{R(h)}^1 \frac{\partial W_s(h, z)}{\partial h} dG(z) - \lambda \frac{\partial R(h)}{\partial h} [W_s(h, R) - U] g(R).$$

Given that Nash bargaining implies $[W_s(h, R) - U] = 0$, this condition simplifies as

$$(r + \lambda)C'(h) = \frac{dw_j(h)}{dh} + \lambda \int_{R(h)}^1 \frac{\partial W_s(h, z)}{\partial h} dG(z).$$

Given the formulation in (26), one can calculate

$$(r + \lambda) \frac{\partial W_s(h, z)}{\partial h} = \frac{dw_s(h, z)}{dh} + \lambda \int_{R(h)}^1 \frac{\partial W_s(h, z)}{\partial h} dG(z)$$

Similarly,

$$(r + \lambda G(R)) \int_R^1 \frac{\partial W_s(h, z)}{\partial h} dG(z) = \int_R^1 \frac{dw_s(h, z)}{dh} dG(z).$$
Figure 6: The impact of a firing tax ($T$) with elastic human capital supply

Note: physical capital, employment, the ratio of physical capital to labor, and human capital are all normalized to one for a $T$ value of zero.

The equation above together with the wage equation

$$w_s(z, h) = (1 - \beta) r U + \beta r T + \beta \Omega_s z \frac{\partial F(J, hZ, K)}{\partial (hZ)},$$

(67)

where

$$\Omega_s = \int_0^1 \frac{1}{\pi} x^{\frac{1-\beta}{\beta}} \frac{\partial F(Jx, hZx, K)}{\partial (hZx)} dx,$$

(68)

give the condition (32).

A.3.2 Numerical exercise, other elasticities for the human capital supply function

In this Appendix, we illustrate how the comparative statics in Section (5) may change when one considers alternative values for the human capital supply function. Specifically, we are interested in understanding under which circumstances, the U-shaped pattern may not be appear anymore.
Figure 7: The impact of a firing tax \((T)\) with low elasticity of supply of human capital

![Graphs showing the impact of firing tax on physical capital, employment, etc.]

Note: physical capital, employment, the ratio of physical capital to labor, and human capital are all normalized to one for a \(T\) value of zero.

The hump-shaped pattern of capital does not appear in the two more extreme situations where the supply elasticity is either large or low, as in Figures 6 and 7 respectively. In Figure 6, which shows the case of an infinitely elastic supply of human capital (i.e. the cost function is linear), the reservation productivity is independent of \(T\) and the stock of human capital is a decreasing function of it. In the Appendix A.3.3, we formally show that this result always hold when the supply of human capital is infinitely elastic. The intuition is the following. When \(T\) increases, the productivity of the least productive job has to decrease. In this context, productivity has to be understood as a combination of the idiosyncratic component \(z\) and the stock of human capital. It turns out that when the supply of human capital is very elastic, it is through a change in the latter component that productivity decreases.

Moreover, Figure 6 shows that employment, capital and the capital-labor ratio decrease with \(T\). The first outcome is due to the fact that only the job finding rate is affected by \(T\) in this case, the job separation rate being kept constant since \(R\) is not affected. Second, because employment decreases and workers are less productive, capital has to decrease as well: it is a complementary factor. Third, the fall in the capital-labor ratio is due to the fall in productivity as in Section 4.2. The value for \(\sigma_1\) we consider in this graph is 3.09 (it is the value that yields 6% annual rate of return.
to seniority).

In Figure 7 we consider a low supply elasticity. We fix $\sigma_2$ at 40. This produces an elasticity of 2.56%. The resulting calibrated value for $\sigma_1$ is close to zero. Figure 7 and Figure 6 display similar comparative statics with the exception that employment increases in this specific example. In our simulations, for all of the cases where no hump-shaped pattern appears, we found that the stock of capital and the capital-labor ratio are always negatively affected by an increase in $T$. This is a natural result as we come back to a situation that resembles the one in Section 4.2 in those cases.

A.3.3 The effect of a firing tax when the supply of human capital is perfectly elastic

Here we formally show that, when the production function is the one we consider in the benchmark economy and the cost of investing in human capital is a linear function of $h$, a firing tax is always associated with less human capital, less physical capital, less employment, lower capital-labor ratio, lower tightness and leave the reservation productivity $R$ unaffected.

Define

$$\hat{k} \equiv \frac{K}{J + hZ} = \frac{K}{N \xi_j + (1 - \xi_j)h\bar{z}_R},$$

the stock of capital per effective worker, where $\xi_j$ is the share of junior workers in employment and $\bar{z}_R$ is the average value of $z$ among senior workers, increasing functions of $R$.

From (31), one can obtain $\hat{k}$ immediately. It is independent of $T$. This is because $F$ is homogeneous of degree one in two arguments, physical capital $K$ and effective workers $(J + hZ)$.

Given a value for $\hat{k}$, (32) allows to get $R$. Hence, $R$ also is independent of $T$. The independence of $R$ implies that $\xi_j$ and $\bar{z}_R$ do not depend on $T$ either.

The rest of the analysis resembles Mortensen and Pissarides (1999), but instead of considering the space $(\theta, R)$ for the job creation and destruction relations, one considers the space $(\theta, h)$. The job creation relation is decreasing in this space and the job destruction is increasing. A firing tax unambiguously decreases $h$ and $\theta$ in the same manner as it decreases $R$ and $\theta$ in Mortensen and Pissarides (1999).

As a consequence, the effect on employment is negative because $\theta$ decreases and $R$ is not affected. Given $\hat{h}$ is independent of $T$ and both $N$ and $h$ decrease for an increase in $T$, $K$ has to decrease as well.

From the definition of $\hat{h}$ above, the capital-labor ratio has to decrease because $h$ decreases.